Communication aspects of a three-player Prisoner's Dilemma quantum game

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Abstract
We present a quantization scheme for a three-player Prisoner's Dilemma game.
It is shown that entanglement plays a dominant role in the three-player quantum
game. Four different types of payoffs are identified on the basis of different
combinations of initial state and measurement basis entanglement parameters.
A relation among these different payoffs is also established. We also study
the communication aspects of the three-player game. By exploiting different
combinations of initial state and measurement basis entanglement parameters,
we establish a relationship for the information shared among the parties. It is
seen that the strategies of the players act as carriers of information in quantum
games.

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1. Introduction

Recent development in quantum computation and quantum information theory [1, 2] prompted
the scope of game theory to extend it to the quantum world. Meyer [3] discussed a connection
between quantum games and quantum information processing. Most of the research on
quantum games has lacked a direct connection to quantum information processing. Quantum
game theory has been extensively studied by a number of authors in recent years [4–6]. The
role of the initial quantum state entanglement is an interesting feature in quantum games.
However, the importance of the payoff operators used by the arbiter to perform measurement
is also important as addressed in [7]. The authors have investigated the role of measurement
basis in quantum games by taking the two-player Prisoner's Dilemma game as an example.
Lee *et al* [8] have studied the problem of quantum state estimation and quantum cloning using
a game-theoretic perspective.

The Prisoner's Dilemma is a widely known example in classical game theory. The study
of multi-player quantum games could be of great importance from both a theoretical and

a practical point of view, and can exhibit interesting results in comparison to two player games. A model of a two-player Prisoner's Dilemma quantum game was developed by Eisert [9] in which the paradox in the classical Prisoner's Dilemma was solved in a maximally entangled state. Quantum Prisoner's Dilemma has been experimentally demonstrated by using a nuclear magnetic resonance (NMR) quantum computer [10]. Recently, Prevedel *et al* have experimentally demonstrated the application of a measurement-based protocol to realize a quantum version of the Prisoner's Dilemma based on entangled photonic cluster states and constituted the first realization of a quantum game in the context of one-way quantum computing [11]. The investigations of multi-player and multi-choice quantum games [12–14] and continuous-variable quantum games [15] have also been pursued in recent years.

With recent interest in quantum computing and quantum information theory, we explore that quantum game theory may be useful for studying the quantum communication, since it can be considered as a game where the objective is to maximize the effective communication. Motivated from our recent paper on two player quantum games [16], we extend our work here to the case of a three-player Prisoner's Dilemma quantum game with the measuring basis taken as entangled. Motivation of a three-player Prisoner's Dilemma quantum game is that more information can be carried by each party which may increase the communication of information. Furthermore, this work may provide a better insight into the study of quantum games from the quantum information and quantum communication perspective. Based on the work discussed in [17, 18], we have attempted to relate the quantum game theory with quantum information theory by investigating the communication aspects of a three-player Prisoner's Dilemma quantum game. Kawakami [17] has studied the communication and information carriers in quantum games. He has shown that communications in quantum games can be used to solve problems that cannot be solved by using communications in classical games.

In this paper, we present a quantization scheme for the three-player Prisoner's Dilemma game using an entangled measuring basis. We study the communication aspects of the three-player Prisoner's Dilemma game by using the players' returns. Based on the flow of information (communication) between players, as evident from the payoff matrix, we establish a relationship for information shared among the parties, for different combinations of initial state and measurement basis entanglement parameters $\gamma \in [0, \pi/2]$ and $\delta \in [0, \pi/2]$ respectively. Here, $\delta = 0$ means that the measurement basis is unentangled, i.e. in a product form and $\delta = \pi/2$ means that it is maximally entangled. Similarly, $\gamma = 0$ means that the game is initially unentangled and $\gamma = \pi/2$ means that it is maximally entangled. We show that the strategies of the players and their payoffs act as information carriers between the players. We establish a relationship among different payoffs on the basis of different combinations of initial state and measurement basis entanglement parameters δ and γ respectively, as studied in [7]. The relation among different quantum payoffs is similar to the relation among classical capacities of the quantum channels [19]. In addition, we also establish a relationship among the information shared between the parties for different combinations of initial state and measurement basis entanglement parameters.

2. The three-player quantization scheme

The three-player Prisoner's Dilemma game is similar to the two-player situation. In a three-player Prisoner's Dilemma game, the players are arrested under the suspicion of committing a crime, say, robbing a bank. Similar to the two-player game, they are interrogated in separate cells without communicating with each other. The two possible moves for each prisoner are, to cooperate (*C*) or to defect (*D*). The payoff table for the three-player Prisoner's Dilemma game is shown in table 1 [13]. The game is symmetric for the three players, and the strategy

Table 1. The payoff matrix for three-player Prisoner's Dilemma where the first number in the parenthesis denotes the payoff of Alice, the second number denotes the payoff of Bob and the third number denotes the payoff of Charlie.

		Charlie C				Charlie D	
		Bob				Bob	
		C	D			C	D
Alice	C	(3, 3, 3)	(2, 5, 2)	Alice	C	(2, 2, 5)	(0, 4, 4)
	D	(5, 2, 2)	(4, 4, 0)		D	(4, 0, 4)	(1, 1, 1)

(D) dominates the strategy (C) for all the three players. Since the selfish players prefer to choose (D) as an optimal strategy, the unique Nash equilibrium is (D, D, D) with payoffs (1, 1, 1). This is a Pareto inferior outcome, since (C, C, C) with payoffs (3, 3, 3) would be better for all three players. This situation is the very catch of the dilemma and is the same as the two-player version of this game.

In our scheme, Alice, Bob and a third player, Charlie, join the game. In this game, an arbiter prepares an initial quantum state and passes it on to the players. After applying their strategies, the players return the state to the arbiter who then announces the payoffs by performing a measurement. Let us suppose that the initial quantum state shared between the three prisoners, consistent with [16, 20], is of the form

$$|\psi_{\text{in}}\rangle = \cos\frac{\gamma}{2}|000\rangle + i\sin\frac{\gamma}{2}|111\rangle, \tag{1}$$

where $0 \leqslant \gamma \leqslant \pi/2$ corresponds to the entanglement of the initial state. Here in this case players can locally manipulate their individual qubits. The possible outcomes of the classical strategies (C) and (D) are assigned the two basis vectors $|0\rangle$ and $|1\rangle$ in the Hilbert space. The strategies of the players can be represented by the unitary operator U_k as defined in [16]

$$U_k = \cos\frac{\theta_k}{2}R_k + \sin\frac{\theta_k}{2}P_k, \tag{2}$$

where $k = A, B$ and C correspond to Alice, Bob and Charlie respectively and R_k, P_k are the unitary operators defined as

$$
\begin{aligned}
R_A|0\rangle &= e^{i\alpha_A}|0\rangle & R_A|1\rangle &= e^{-i\alpha_A}|1\rangle \\
P_A|0\rangle &= e^{i(\frac{\pi}{2}-\beta_A)}|1\rangle & P_A|1\rangle &= e^{i(\frac{\pi}{2}+\beta_A)}|0\rangle \\
R_B|0\rangle &= e^{i\alpha_B}|0\rangle & R_B|1\rangle &= e^{-i\alpha_B}|1\rangle \\
P_B|0\rangle &= e^{i(\frac{\pi}{2}-\beta_B)}|1\rangle & P_B|1\rangle &= e^{i(\frac{\pi}{2}+\beta_B)}|0\rangle \\
R_C|0\rangle &= e^{i\alpha_C}|0\rangle & R_C|1\rangle &= e^{-i\alpha_C}|1\rangle \\
P_C|0\rangle &= e^{i(\frac{\pi}{2}-\beta_C)}|1\rangle & P_C|1\rangle &= e^{i(\frac{\pi}{2}+\beta_C)}|0\rangle,
\end{aligned}
\tag{3}
$$

where $0 \leqslant \theta_k \leqslant \pi, -\pi \leqslant \{\alpha_k, \beta_k\} \leqslant \pi$. By the application of the local operators of the players, the initial state given in equation (1) transforms to

$$\rho_f = (U_A \otimes U_B \otimes U_C)\rho_{\text{in}}(U_A \otimes U_B \otimes U_C)^\dagger, \tag{4}$$

where $\rho_{\text{in}} = |\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|$ is the initial density matrix for the quantum state. The operators used by the arbiter to determine the payoffs for Alice, Bob and Charlie are

$$
\begin{aligned}
P^k = {}&\$^k_{000}P_{000} + \$^k_{001}P_{001} + \$^k_{110}P_{110} + \$^k_{010}P_{010} \\
&+ \$^k_{101}P_{101} + \$^k_{011}P_{011} + \$^k_{100}P_{100} + \$^k_{111}P_{111}
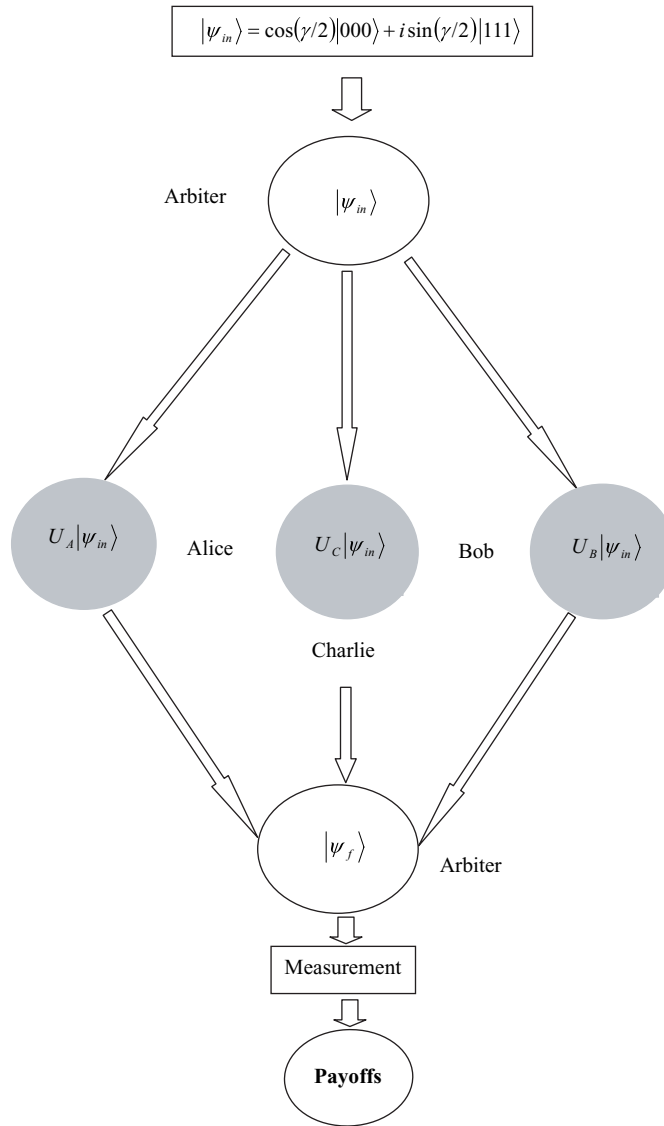\end{aligned}
\tag{5}
$$

where

$$P_{000} = |\psi_{000}\rangle\langle\psi_{000}|, \qquad |\psi_{000}\rangle = \cos\frac{\delta}{2}|000\rangle + \mathrm{i}\sin\frac{\delta}{2}|111\rangle$$

$$P_{111} = |\psi_{111}\rangle\langle\psi_{111}|, \qquad |\psi_{111}\rangle = \cos\frac{\delta}{2}|111\rangle + \mathrm{i}\sin\frac{\delta}{2}|000\rangle$$

$$P_{001} = |\psi_{001}\rangle\langle\psi_{001}|, \qquad |\psi_{001}\rangle = \cos\frac{\delta}{2}|001\rangle + \mathrm{i}\sin\frac{\delta}{2}|110\rangle$$

$$P_{110} = |\psi_{110}\rangle\langle\psi_{110}|, \qquad |\psi_{110}\rangle = \cos\frac{\delta}{2}|110\rangle + \mathrm{i}\sin\frac{\delta}{2}|001\rangle$$

$$P_{010} = |\psi_{010}\rangle\langle\psi_{010}|, \qquad |\psi_{010}\rangle = \cos\frac{\delta}{2}|010\rangle - \mathrm{i}\sin\frac{\delta}{2}|101\rangle \tag{6}$$

$$P_{101} = |\psi_{101}\rangle\langle\psi_{101}|, \qquad |\psi_{101}\rangle = \cos\frac{\delta}{2}|101\rangle - \mathrm{i}\sin\frac{\delta}{2}|010\rangle$$

$$P_{011} = |\psi_{011}\rangle\langle\psi_{011}|, \qquad |\psi_{011}\rangle = \cos\frac{\delta}{2}|011\rangle - \mathrm{i}\sin\frac{\delta}{2}|100\rangle$$

$$P_{100} = |\psi_{100}\rangle\langle\psi_{100}|, \qquad |\psi_{100}\rangle = \cos\frac{\delta}{2}|100\rangle - \mathrm{i}\sin\frac{\delta}{2}|011\rangle$$

where $0 \leqslant \delta \leqslant \pi/2$ and $\$_{lmn}^{k}$ are the elements of the payoff matrix as given in table 1. Since quantum mechanics is a fundamentally probabilistic theory, the strategic notion of the payoff is the expected payoff. The players after their actions that leave the game in a state given in equation (4) forward their qubits to the arbiter for the final projective measurement, for example, in the computational basis as given in equation (6), who determines their payoffs (as shown in figure 1). The payoffs for the players can be obtained as the mean values of the payoff operators

$$\$^{k}(\theta_k, \alpha_A, \beta_A) = \mathrm{Tr}(P^k \rho_f), \tag{7}$$

where Tr represents the trace of the matrix. Using equations (1)–(7), the payoffs of the three players are given by

$$
\begin{aligned}
\$^{k}(\theta_k, \alpha_k, \beta_k) = {} & c_A c_B c_C \left[\eta_1 \$_{000}^{k} + \eta_2 \$_{111}^{k} + \left(\$_{000}^{k} - \$_{111}^{k}\right)\xi \cos 2(\alpha_A + \alpha_B + \alpha_C)\right] \\
& + s_A s_B s_C \left[\eta_2 \$_{000}^{k} + \eta_1 \$_{111}^{k} - \left(\$_{000}^{k} - \$_{111}^{k}\right)\xi \cos 2(\beta_A + \beta_B + \beta_C)\right] \\
& + c_A c_B s_C \left[\eta_1 \$_{001}^{k} + \eta_2 \$_{110}^{k} + \left(\$_{001}^{k} - \$_{110}^{k}\right)\xi \cos 2(\alpha_A + \alpha_B - \beta_C)\right] \\
& + s_A s_B c_C \left[\eta_2 \$_{001}^{k} + \eta_1 \$_{110}^{k} - \left(\$_{001}^{k} - \$_{110}^{k}\right)\xi \cos 2(\beta_A + \beta_B - \alpha_C)\right] \\
& + s_A c_B c_C \left[\eta_1 \$_{100}^{k} + \eta_2 \$_{011}^{k} + \left(\$_{100}^{k} - \$_{011}^{k}\right)\xi \cos 2(\alpha_B + \alpha_C - \beta_A)\right] \\
& + c_A s_B s_C \left[\eta_2 \$_{100}^{k} + \eta_1 \$_{011}^{k} - \left(\$_{100}^{k} - \$_{011}^{k}\right)\xi \cos 2(\beta_B + \beta_C - \alpha_A)\right] \\
& + s_A c_B s_C \left[\eta_1 \$_{101}^{k} + \eta_2 \$_{010}^{k} + \left(\$_{101}^{k} - \$_{010}^{k}\right)\xi \cos 2(\beta_A + \beta_C - \alpha_B)\right] \\
& + c_A s_B c_C \left[\eta_2 \$_{101}^{k} + \eta_1 \$_{010}^{k} - \left(\$_{101}^{k} - \$_{010}^{k}\right)\xi \cos 2(\alpha_A + \alpha_C - \beta_B)\right] \\
& + \tfrac{1}{8}(\cos^2(\delta/2) - \sin^2(\delta/2))\left[\$_{000}^{k} - \$_{111}^{k} - \$_{001}^{k} + \$_{110}^{k} - \$_{010}^{k} + \$_{101}^{k} + \$_{011}^{k}\right. \\
& \left. - \$_{100}^{k}\right]\sin(\gamma)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\alpha_A + \alpha_B + \alpha_C - \beta_A - \beta_B - \beta_C) \\
& + \left[\left[\$_{000}^{k} - \$_{111}^{k}\right]\sin(\delta)\sin(\theta_1)\sin(\theta_2)\sin(\theta_2)\cos(\alpha_A + \alpha_B + \alpha_C\right. \\
& - \beta_A - \beta_B - \beta_C) + \left[\$_{110}^{k} - \$_{001}^{k}\right]\sin(\delta)\sin \\
& \times (\theta_1)\sin(\theta_2)\sin(\theta_2)\cos(\alpha_A + \alpha_B - \alpha_C + \beta_A + \beta_B - \beta_C) \\
& + \left[\$_{010}^{k} - \$_{101}^{k}\right]\sin(\delta)\sin(\theta_1)\sin(\theta_2)\sin(\theta_2)\cos \\
& \times (\alpha_A - \alpha_B + \alpha_C + \beta_A - \beta_B + \beta_C)
\end{aligned}
$$

4

$$\boxed{|\psi_{in}\rangle = \cos(\gamma/2)|000\rangle + i\sin(\gamma/2)|111\rangle}$$

Figure 1. The schematic diagram of the procedure of the game.

$$+ \left[\$_{100}^k - \$_{011}^k\right]\sin(\delta)\sin(\theta_1)\sin(\theta_2)\sin(\theta_2)\cos$$
$$\times (\alpha_A - \alpha_B - \alpha_C + \beta_A - \beta_B - \beta_C)\big]$$
$$\times \left[\tfrac{1}{8}(\cos^2(\gamma/2) - \sin^2(\gamma/2))\right] \tag{8}$$

where

$$\eta_1 = \cos^2(\gamma/2)\cos^2(\delta/2) + \sin^2(\gamma/2)\sin^2(\delta/2)$$
$$\eta_2 = \sin^2(\gamma/2)\cos^2(\delta/2) + \sin^2(\delta/2)\cos^2(\gamma/2)$$
$$\xi = \frac{1}{2}\sin(\delta)\sin(\gamma)$$

Table 2. The payoffs of the three players for $\gamma = \delta = 0$ and $\gamma = \delta = \pi/2$, for different Alice's operations, as obtained from equation (8).

Alice's unitary operation	$U_C(0)$		$U_C(\pi)$	
	$U_B(0)$	$U_B(\pi)$	$U_B(0)$	$U_B(\pi)$
$U_A(0, 0, 0)$	(3, 3, 3)	(2, 5, 2)	(2, 2, 5)	(0, 4, 4)
$U_A(\pi/3, \pi/2, \pi/2)$	(3/4, 7/4, 7/4)	(7/2, 1/2, 17/4)	(7/2, 17/4, 1/2)	(9/2, 9/4, 9/4)
$U_A(\pi/2, \pi/2, \pi/2)$	(1/2, 5/2, 5/2)	(3, 1, 9/2)	(3, 9/2, 1)	(4, 5/2, 5/2)
$U_A(\pi, \pi, \pi)$	(5, 2, 2)	(4, 4, 0)	(4, 0, 4)	(1, 1, 1)

$$c_k = \cos^2 \frac{\theta_k}{2}$$
$$s_k = \sin^2 \frac{\theta_k}{2}. \tag{9}$$

The payoffs for the three players can be found by substituting the appropriate values for $\$^k_{lmn}$ into equation (8). The elements of the classical payoff matrix for the Prisoner's Dilemma game are given in table 1. Our results are consistent with [13] and can easily be checked from equation (8), when all the three players resort to their Nash equilibrium strategies.

3. Communication scenario

Let us start with an analysis of the communication aspects of the quantized Prisoner's Dilemma game. The communication aspect of quantum games is similar to the dense coding [21], in the sense that, we can transmit two bits of classical information by sending only one qubit with the help of entanglement while the sender and the receiver share an entangled quantum state. Motivation of the three-player quantum game is that more information can be carried by each party which may increase the information flux in comparison to the standard two-player version of the Prisoner's Dilemma game. Furthermore, the realization of the communication is due to the advantage of quantum strategies and quantum entanglement. In our approach, each prisoner has his/her private qubit and applies the unitary transformation to this. Their arbiter gives a payoff to each of them based on a measured result of each qubit. Unitary transformations are strategies for prisoners which play a key role in constructing the payoff matrix. Here we consider that the strategies of prisoners are represented by the local operators of Alice, Bob and Charlie as given in equation (2). Let Alice, Bob and Charlie agree on that Alice performs the following four unitary operations out of the set $U_A(\theta_A, \alpha_A, \beta_A)$, as given in the below equation, on her qubit

$$
\begin{aligned}
U_A(0, 0, 0) &\Rightarrow 00 \\
U_A\left(\frac{\pi}{3}, \frac{\pi}{2}, \frac{\pi}{2}\right) &\Rightarrow 01 \\
U_A\left(\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}\right) &\Rightarrow 10 \\
U_A(\pi, \pi, \pi) &\Rightarrow 11
\end{aligned}
\tag{10}
$$

where 00, 01, 10, 11 represent the exchange of two bits of information. In order to obtain the classical payoff matrix, we consider the case of a restricted game, where Alice is allowed to get benefit from the quantum phases whereas Bob and Charlie are restricted to doing so with a fixed phase change by setting $\alpha_B = \alpha_C = 0$ and $\beta_B = \beta_C = \pi/2$. Thus, restricting Bob and Charlie to only applying $\theta_{B(C)} = 0$ or π as their set of strategies, utilizing which one can

construct the classical payoff matrix for the three-player Prisoner's Dilemma game as given in table 1. As a result of measurement, Bob and Charlie can extract the information about the strategy applied by Alice from their payoffs by a mutual understanding that they will apply the same strategy, i.e. either $\theta_{B(C)} = 0$ or $\theta_{B(C)} = \pi$, such a cooperation between the users can avoid corruption in quantum communication. Because, the application of the unitary operators changes not only the value of a qubit, but also its phase (amplitude). This results in a communication of two bits of information by two local one-qubit operations among the parties (as seen from table 2).

For example, let Bob and Charlie apply $\theta_{B(C)} = 0$, and gain the payoffs 2, 2 respectively and they can easily find that the decision of Alice was $U_A(\pi, \pi, \pi)$ with payoff 5 as can be seen from table 2. In this case, information which is exchanged between them through the arbiter is represented as 2 bits, to determine one of the four possibilities.

4. Relationship between payoffs and information

Quantum payoffs can be divided into four different categories on the basis of four different combinations of the initial state and measurement basis entanglement parameters γ and δ. These different situations arise due to the possibility of having a product or entangled initial state and then applying a product or entangled basis for the measurement [22, 23]. Here, we will use the subscripts E and P which correspond to the entangled and product basis being used for quantum payoffs respectively. The four different types of payoffs can be categorized as

Case (a). When $\delta = \gamma = 0$ (i.e. the initial quantum state used is in the product form, and the product basis is used for measurement to determine the payoffs), the game becomes classical and each player plays the strategy C, with probability $\cos^2(\theta_k/2)$ and the payoffs for the players at the Nash equilibrium become

$$\$_{PP}^k(\theta_k = \pi) = 1. \tag{11}$$

Case (b). When $\gamma = 0, \delta \neq 0$ (i.e. the initial quantum state used is in the product form, and the entangled basis is used for measurement to determine the payoffs) the players' payoff remains less than 3 at the two Nash equilibria arising at $\theta_k = 0$ and $\pi/2$ which reads

$$\$_{PE}^k(\theta_k = \pi/2, \alpha_A = \pi, \beta_A = \pi) < 3 \tag{12}$$

$$\$_{PE}^k(\theta_k = 0, \alpha_A = \pi, \beta_A = \pi) < 3. \tag{13}$$

Case (c). When $\delta = 0, \gamma \neq 0$ (i.e. the initial quantum state is entangled, and the product basis is used for measurement to determine the payoffs), the players' payoff again remains less than 3 at the two Nash equilibria and is given as

$$\$_{EP}^k(\theta_k = \pi/2, \alpha_A = \pi, \beta_A = \pi) < 3 \tag{14}$$

$$\$_{EP}^k(\theta_k = 0, \alpha_A = \pi, \beta_A = \pi) < 3. \tag{15}$$

Case (d). When $\gamma = \delta = \pi/2$ (i.e. the initial quantum state is in an entangled form and the entangled basis is used for measurement to determine the payoffs), the players' payoff can be obtained from

Table 3. The payoffs of the three players for $\gamma = 0, \delta = \pi/2$ and $\gamma = \pi/2, \delta = 0$, for different Alice's operations, as obtained from equation (8).

Alice's unitary operation	$U_C(0)$		$U_C(\pi)$	
	$U_B(0)$	$U_B(\pi)$	$U_B(0)$	$U_B(\pi)$
$U_A(0, 0, 0)$	(2, 2, 2)	(3, 5/2, 3)	(3, 3, 5/2)	(5/2, 3, 3)
$U_A(\pi/3, \pi/2, \pi/2)$	(17/8, 9/4, 9/4)	(3, 21/8, 23/8)	(3, 23/8, 21/8)	(19/8, 11/4, 11/4)
$U_A(\pi/2, \pi/2, \pi/2)$	(9/4, 5/2, 5/2)	(3, 11/4, 11/4)	(3, 11/4, 11/4)	(9/4, 5/2, 5/2)
$U_A(\pi, \pi, \pi)$	(5/2, 3, 3)	(3, 3, 5/2)	(3, 5/2, 3)	(2, 2, 2)

$$
\begin{aligned}
\$_{EE}^k(\theta_k, \alpha_A, \beta_A) = & \frac{c_A c_B c_C}{2}\left[\left(\$_{000}^k + \$_{111}^k\right) + \left(\$_{000}^k - \$_{111}^k\right)\xi \cos 2(\alpha_A)\right] \\
& + \frac{s_A s_B s_C}{2}\left[\left(\$_{000}^k + \$_{111}^k\right) - \left(\$_{000}^k - \$_{111}^k\right)\xi \cos 2(\beta_A)\right] \\
& + \frac{c_A c_B s_C}{2}\left[\left(\$_{001}^k + \$_{110}^k\right) + \left(\$_{001}^k - \$_{110}^k\right)\xi \cos 2(\alpha_A)\right] \\
& + \frac{s_A s_B c_C}{2}\left[\left(\$_{001}^k + \$_{110}^k\right) - \left(\$_{001}^k - \$_{110}^k\right)\xi \cos 2(\beta_A)\right] \\
& + \frac{s_A c_B c_C}{2}\left[\left(\$_{100}^k + \$_{011}^k\right) + \left(\$_{100}^k - \$_{011}^k\right)\xi \cos 2(\beta_A)\right] \\
& + \frac{c_A s_B s_C}{2}\left[\left(\$_{100}^k + \$_{011}^k\right) - \left(\$_{100}^k - \$_{011}^k\right)\xi \cos 2(\alpha_A)\right] \\
& + \frac{s_A c_B s_C}{2}\left[\left(\$_{101}^k + \$_{010}^k\right) + \left(\$_{101}^k - \$_{010}^k\right)\xi \cos 2(\beta_A)\right] \\
& + \frac{c_A s_B c_C}{2}\left[\left(\$_{101}^k + \$_{010}^k\right) - \left(\$_{101}^k - \$_{010}^k\right)\xi \cos 2(\alpha_A)\right].
\end{aligned} \tag{16}
$$

The payoffs when the three players play their Nash equilibrium strategies become

$$
\$_{EE}^k(\theta_k = 0, \alpha_A = \pi, \beta_A = \pi) = 3. \tag{17}
$$

From the above four cases one can establish the following relation among the four payoff values as

$$
\$_{PP}^k < \$_{PE}^k = \$_{EP}^k < \$_{EE}^k \tag{18}
$$

at the Nash equilibrium.

Furthermore, for the above four cases, we construct the payoff matrix as obtained from equation (8) for Alice's four unitary operations as given in equation (10). For $\gamma = \delta = 0$ and $\gamma = \delta = \pi/2$, the payoff matrix can be obtained from equation (8) as given in table 2; whereas for $\gamma = 0, \delta = \pi/2$ and $\gamma = \pi/2, \delta = 0$, the payoff matrix can be obtained from equation (8) as given in table 3. We can determine the payoff which is given to each prisoner on the basis of his strategy from equation (8). We can see from equations (3) and (10) that each strategy can be distinguished from the set $U_A(\theta_A, \alpha_A, \beta_A)$, $U_B(\theta_B)$ and $U_C(\theta_C)$. It is assumed that the two parties Bob and Charlie have a mutual agreement with each other that they will apply the same strategy in order to find out the strategy applied by Alice. Let Bob and Charlie apply $\theta_{B(C)} = \pi$, and gain the payoffs 4, 4 respectively, then they can find that the decision of Alice was $U_A(0, 0, 0)$ with payoff 0 as seen from table 2. In this way, they can find all the four strategies applied by Alice from their payoffs, which results in an information exchange between the parties through the arbiter. However, for $\gamma = 0, \delta = \pi/2$ and $\gamma = \pi/2, \delta = 0$, half of the information is lost because the phase information vanishes due to the overlapping of half of the entries of the payoff matrix as seen from table 3. So there is one half probability of finding out exactly the strategy applied by Alice.

Therefore, from tables 2 and 3 we see that Bob and Charlie can find the unitary operators applied by Alice from their payoffs against their common strategy. As a result, there is a communication of two bits of information by two local one-qubit operations among the three parties (as seen from table 2). However, we can see from table 3 that the information shared between the parties is halved because there is one half probability of finding the exact strategy of Alice. Thus, we can establish a relationship among the amounts of information communicated between the parties as

$$\{I_{PP} = I_{EE}\} > \{I_{PE} = I_{EP}\} \tag{19}$$

The above relation holds for the set of Alice's four unitary operations under the bound that Bob and Charlie are restricted to play a common move.

5. Conclusion

We present a quantization scheme for the three-player Prisoner's Dilemma game using entangled measuring basis. We show that entanglement plays a dominant role in a three-player quantum game. We study the communication aspects of a three-player quantum game which is similar to the dense coding where two bits of classical information can be transmitted by the sender. It is seen that three-player quantum games are advantageous in the sense that more information can be carried by the players, thus enhancing the information flux in comparison to the two-player games. It can be seen that the communication is due to the advantage of quantum entanglement and quantum strategies. We investigate that the strategies of the players act as information carriers in quantum games. We identify four different payoffs on the basis of different combinations of initial state and measurement basis entanglement parameters. A relation among these different payoffs is also established. Exploiting different combinations of initial state and measurement basis entanglement parameters, we establish a relationship for the information shared among the parties.

References

[1] Nielson M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Bouwmeester D, Ekert A and Zeilinger A 2000 *The Physics of Quantum Information* (Berlin: Springer)
[3] Meyer D A 1999 *Phys. Rev. Lett.* **82** 1052
[4] Flitney A P and Abbott D 2002 *Fluct. Noise Lett.* **2** R175
[5] Lee C F and Johnson N 2003 *Phys. Rev.* A **67** 022311
[6] Du J *et al* 2003 *J. Phys. A: Math. Gen.* **36** 6551
[7] Nawaz A and Toor A H 2006 *J. Phys. A: Math. Gen.* **39** 2791
[8] Lee C F and Johnson N F 2003 *Phys. Lett.* A **319** 429
[9] Eisert J, Wilkens M and Lewenstein M 1999 *Phys. Rev. Lett.* **83** 3077
[10] Du J, Li H, Xu X, Shi M, Wu J, Zhou X and Han R 2002 *Phys. Rev. Lett.* **88** 137902
[11] Prevedel R, Stefanov A, Walther P and Zeilinger A 2007 *New J. Phys.* **9** 205
[12] Benjamin S C and Hayden P M 2001 *Phys. Rev.* A **64** 030301
[13] Du J *et al* 2002 *Phys. Lett.* A **302** 229
[14] Flitney A P and Abbott D 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** S860
[15] Li H, Du J F and Massar S 2002 *Phys. Lett.* A **306** 73
[16] Ramzan M, Nawaz A, Toor A H and Khan M K 2008 *J. Phys. A: Math. Theor.* **41** 055307
[17] Kawakami T 2002 *Lecture Notes in Computer Science* (Berlin: Springer)
[18] Grabbe J O 2005 arXiv:quant-ph/0506219
[19] King C and Ruskai M B 2001 *J. Math. Phys.* **42** 87
[20] Nawaz A and Toor A H 2006 *J. Phys. A: Math. Gen.* **37** 11457
[21] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[22] Pati A K and Agrawal P 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** S844
[23] Kim Y H, Kulik S P and Shih Y 2001 *Phys. Rev. Lett.* **86** 1370